Differentialgleichung der Wärmeleitung um eine Kugel

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> restart :
> DGL := diff(T(r,t),t) = a·(diff(diff(T(r,t),r),r) + 2/r·diff(T(r,t),r)) + W/(rho·cp);
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$$DGL := \frac{\partial}{\partial t} T(r,t) = a\left(\frac{\partial^2}{\partial r^2} T(r,t) + \frac{2\left(\frac{\partial}{\partial r} T(r,t)\right)}{r}\right) + \frac{W}{\rho\, cp} \tag{1}$$

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> LOES := pdsolve(DGL, T(r,t), build);
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$$LOES := T(r,t) = \frac{\_C3\, e^{a\_c_1\, t}\, \_C1 \sinh\left(\sqrt{\_c_1}\, r\right)}{r} + \frac{\_C2\, \_C3\, e^{a\_c_1\, t} \cosh\left(\sqrt{\_c_1}\, r\right)}{r} - \frac{1}{6}\frac{W\_C1}{a\,\rho\, cp\, \_C2} - \frac{1}{6}\frac{r^2 W}{a\,\rho\, cp} - \frac{1}{6}\frac{W\_C3}{r\, a\,\rho\, cp\, \_C2} \tag{2}$$

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> T1 := W·(Rs^2 - r^2 + 2·Rs^2·λ1/λ2)/(6·λ1); T2 := W·Rs^3/(3·λ2·r);
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$$T1 := \frac{1}{6}\frac{W\left(Rs^2 - r^2 + \frac{2\, Rs^2 \lambda_1}{\lambda_2}\right)}{\lambda_1}$$

$$T2 := \frac{1}{3}\frac{W\, Rs^3}{\lambda_2\, r} \tag{3}$$

```
> DGL1 := subs(T(r,t) = T1, DGL);
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$$DGL1 := \frac{\partial}{\partial t}\left(\frac{1}{6}\frac{W\left(Rs^2 - r^2 + \frac{2\, Rs^2 \lambda_1}{\lambda_2}\right)}{\lambda_1}\right) = a\left(\frac{\partial^2}{\partial r^2}\left(\frac{1}{6}\frac{W\left(Rs^2 - r^2 + \frac{2\, Rs^2 \lambda_1}{\lambda_2}\right)}{\lambda_1}\right) + \frac{2\left(\frac{\partial}{\partial r}\left(\frac{1}{6}\frac{W\left(Rs^2 - r^2 + \frac{2\, Rs^2 \lambda_1}{\lambda_2}\right)}{\lambda_1}\right)\right)}{r}\right) + \frac{W}{\rho\, cp} \tag{4}$$

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> DGL2 := subs(T(r,t) = T2, subs(W = 0, DGL));
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$$DGL2 := \frac{\partial}{\partial t}\left(\frac{1}{3}\,\frac{W Rs^3}{\lambda_2 r}\right) = a\left(\frac{\partial^2}{\partial r^2}\left(\frac{1}{3}\,\frac{W Rs^3}{\lambda_2 r}\right) + \frac{2\left(\frac{\partial}{\partial r}\left(\frac{1}{3}\,\frac{W Rs^3}{\lambda_2 r}\right)\right)}{r}\right) \tag{5}$$

> $expand(\%);$

$$0 = 0 \tag{6}$$

> $expand(DGL1);$

$$0 = -\frac{a W}{\lambda_1} + \frac{W}{\rho\, cp} \tag{7}$$

> $Tges := piecewise(r < Rs, T1, r > Rs, T2);$

$$Tges := \begin{cases} \frac{1}{6}\,\frac{W\left(Rs^2 - r^2 + \frac{2\,Rs^2\,\lambda_1}{\lambda_2}\right)}{\lambda_1} & r < Rs \\ \frac{1}{3}\,\frac{W Rs^3}{\lambda_2 r} & Rs < r \end{cases} \tag{8}$$

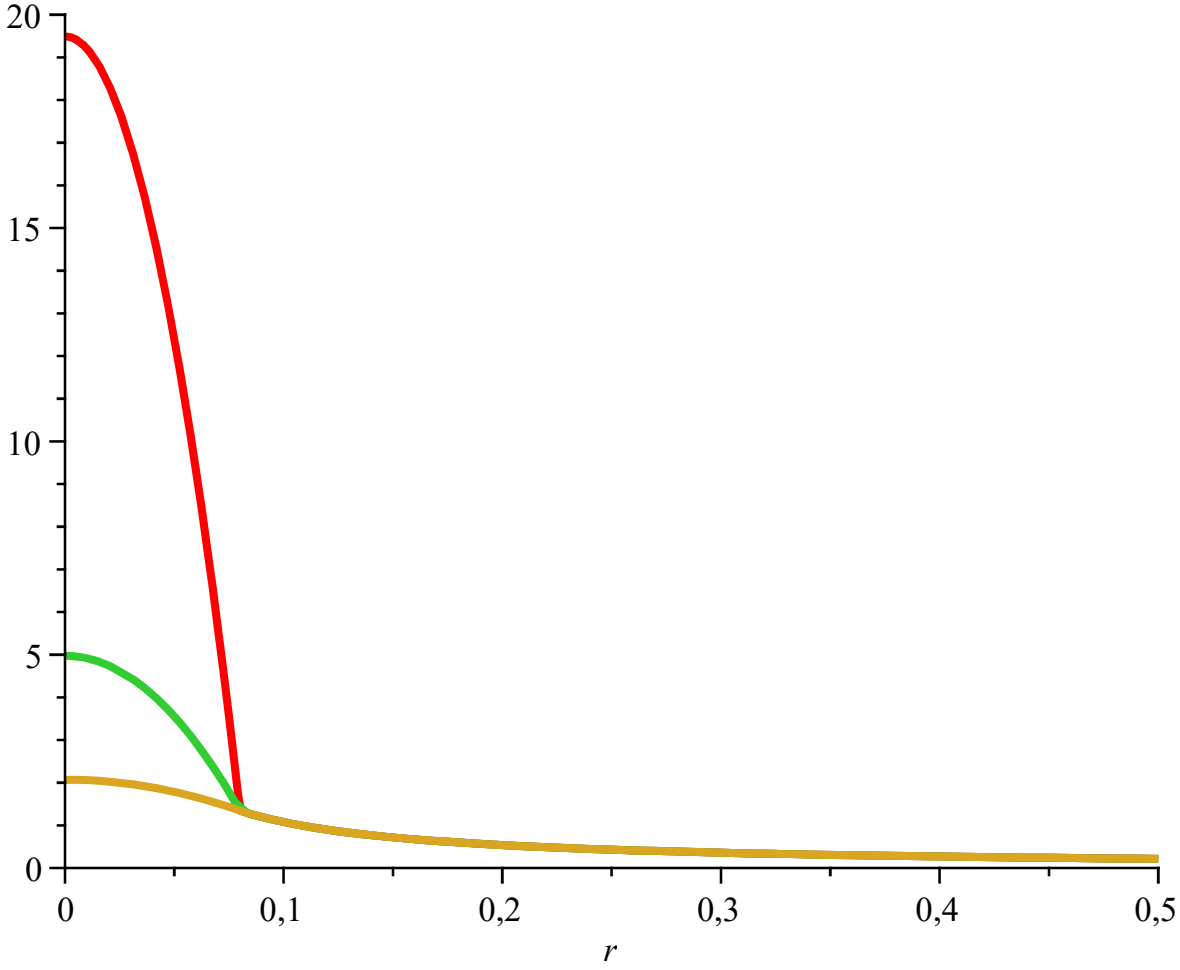
> $Volumen := \frac{Pi}{4}\cdot 0.08^2\cdot(0.192 + 0.06);$

$$Volumen := 0.0004032000000\,\pi \tag{9}$$

> $Q := 4.31;\ W := \frac{Q}{Volumen};$

$$Q := 4.31$$
$$W := \frac{10689.48413}{\pi} \tag{10}$$

> $plot(\{subs(\lambda_1 = 0.2, \lambda_2 = 5.4, Rs = 0.08, Tges), subs(\lambda_1 = 1, \lambda_2 = 5.4, Rs = 0.08, Tges), subs(\lambda_1 = 5, \lambda_2 = 5.4, Rs = 0.08, Tges)\}, r = 0..0.5, 0..20);$



> $evalf\left(subs\left(\lambda_1 = 15, \lambda_2 = 5.4, Rs = 0.08, Tges\right)\right);$

$$\begin{cases} 1.586185008 - 37.80631640\, r^2 & r < 0.08 \\ \dfrac{0.1075379667}{r} & 0.08 < r \end{cases} \qquad \textbf{(11)}$$

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